One Model Doesn't Fit All: Recent Results of a Detailed Analysis of Sunspot Demographics

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SUNSPOTS AND THE SOLAR CYCLE
Sunspots are the optical signature of the presence of strong magnetic fields. They are generally associated with tilted bipolar magnetic regions.
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BMRs are critical for cycle propagation

Poloidal: $r - \theta$

Toroidal: $\phi$

Polar Fields

Sunspot Numbers/Area

Credit: J. J. Love
BMRs are critical for cycle propagation


We used optical proxies to study the connection between toroidal and poloidal fields

Poloidal

Toroidal

$r - \theta$

$\phi$

Sunspot Area (mHem)

Signed Polar Flux (10^22 Mx)

Year


North

South

North

South
BMRs are critical for cycle propagation


The relationship between polar fields at minimum and cycle amplitude likes at the core of most physics based cycle predictions.
BMRs are critical for cycle propagation


Poloidal \( r - \theta \)

Differential Rotation

Toroidal \( \phi \)

Emergence and Decay of Tilted Active Regions
BMRs are critical for cycle propagation

Poloidal $r - \theta$

Toroidal $\phi$

Differential Rotation

Emergence and Decay of Tilted Active Regions

\[
\text{Amplitude Next Cycle (mHem)} = \rho \times \text{Average Polar Flux During Min (10^{22} Mx)}
\]

\[
\text{Avg. Polar Flux During Min (10^{22} Mx)} = \rho \times \text{Max. Cycle A. * AWNL Avg. Tilt (mHem)}
\]

\[
\text{rho} = 0.6, \ pval = 99\%
\]

\[
\text{rho} = 0.74, \ pval = 99\%
\]
BMRs are critical for cycle propagation


Poloidal \( r - \theta \) \[ \rightarrow \] \[ \leftarrow \] Toroidal \( \phi \)

Differential Rotation

Emergence and Decay of Tilted Active Regions

• Our results are consistent with this picture of the solar cycle.

• BMR and sunspot properties are highly variable statistical characterizations are necessary.
Our Data

• Sunspot Group Area:
  – SDO/HMI. 2010 - present.

• Sunspot Area:
  – SDO/HMI Umbral. 2010 - present.

• Bipolar Magnetic Region Flux:
  – KPVT/SOLIS. 1996 - present.
• Structures near the lower detection threshold suffer from a host of issues that can potentially distort our statistical analysis.
Data Truncation

- To avoid this issue, we impose a truncation limit one order of magnitude above the minimum size of detection.
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Only data inside dark areas is included in our fits and analysis, light areas are shown for visual reference.
AREA AND FLUX DISTRIBUTION

Muñoz-Jaramillo et al., *ApJ*, **800**:48, **2015**

In collaboration with Ryan Senkpeil, John Windmueller, Ernest Amouzou, Dana Longcope, Andrey Tlatov, Yury Nagovitsyn, Alexei Pevtsov, Gary Chapman, Angela Cookson, Anthony Yeates, Fraser Watson, Laura Balmaceda, Piet Martens, & Ed DeLuca
Which distribution to use?

**Power Law**
- Zharkov et al. (2005)
- Meunier (2003)
- Hagenaar et al. (2003)
- Parnell et al. (2009)

**Log-Normal**
- Bogdan et al. (1988)
- Baumann & Solanki (2005)
- Zhang et al. (2010)
- Schad & Penn (2010)

**Weibull**
- Parnell (2002)

**Exponential**
- Tang et al. (1984)
- Schrijver et al. (1997)

**Composite Distributions**
- Kuklin (1980)
- Harvey & Zwaan (1993)
- Jiang et al. (2011)
- Nagovitsyn et al. (2012)
Which distribution to use?

- We fitted our 11 databases with these four distributions (power-law, log-normal, Weibull, exponential), and a log-normal-Weibull composite.

- We applied a quantitative model selection criterion called Akaike’s Information Criterion (AIC; Akaike 1983):

  \[ AIC = -2 \ln(\hat{M}) - 2n \]

- In AIC, the model’s log-likelihood (\(\ln\)) and the fitted model’s degrees of freedom (\(n\)) are used to strike a balance between underfitting and overfitting.

- AIC is a relative method of model discrimination, the BEST model is not necessarily the “TRUE” model.
Single Fits

Sunspot Group Area

Sunspot Group Area RGO

Sunspot Group Area SOON

Sunspot Group Area KMAS

Sunspot Group Area PCSA

Sunspot Group Area HMI

Sunspot Umbra Area MDI

Sunspot Umbra Area HMI

Sunspot Area SFO

BMRs KPVT

BMRs KPVT/SOLIS

Sunspot Area

BMRs MDI

BMR Flux
Single Fits

Better Fitted by Weibull

Better Fitted by Log-Normal
A combination of Weibull and Log-normal distributions fits the data best.

• Different sets are sampling different sections of a single distribution.

• Conflicting results arise from different data types sampling different part of this distribution.
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- The log-normal component is populated by the largest sunspot groups/BMRs, the Weibull component by small pores.
CYCLE DEPENDENCE OF SUNSPOT GROUP PROPERTIES


In collaboration with Ryan Senkpeil, Dana Longcope, Andrey Tlatov, Alexei Pevtsov, Piet Martens, & Ed DeLuca
• Cycle evolution of active regions and sunspots is normally studied by comparing separate cycles or phases (minimum vs. maximum).

• This approach is sub-optimal for studying sunspot and BMR properties. Why?
The lifetime of an active region is but an instant compared with the cycle.

Assumption: The global properties of a cycle are irrelevant for determining the properties of active regions. Only activity level is important.
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Assumption: The global properties of a cycle are irrelevant for determining the properties of active regions. Only activity level is important.
Statistical properties associated with low activity levels are observed in every cycle. Statistical properties can be different in each hemisphere.
Activity Level and the empirical distribution function

- There is a very clear dependence of the relative amount of large sunspot groups and higher activity levels.
- The Weibull-Log-Normal composite captures successfully this variation.
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The Weibull-Log-Normal composite captures successfully this variation.
Activity level and the analytic probability density function

\[ f(x \mid k, \lambda, \mu, \sigma, c) = \left(1 - c\right) \frac{k}{\lambda} \left(\frac{x}{\lambda}\right)^{k-1} e^{-\left(\frac{x}{\lambda}\right)} + \frac{c}{x \sigma \sqrt{2\pi}} e^{-\frac{(\ln x - \mu)^2}{2\sigma^2}} \]

**Weibull**

- Scale Factor \((\lambda)\)
- Shape Parameter \((k)\)

**Log-Normal**

- Mean \((\mu)\)
- Variance \((\sigma)\)
Activity level and the analytic probability density function

\[ f(x \mid k, \lambda, \mu, \sigma, c) = (1 - c) \frac{k}{\lambda} \left( \frac{x}{\lambda} \right)^{k-1} e^{-\left( \frac{x}{\lambda} \right)^k} + \frac{c}{x \sigma \sqrt{2\pi}} e^{-\frac{(\ln x - \mu)^2}{2 \sigma^2}} \]

• The Weibull’s parameters are not correlated activity level (Hagenaar et al 2003, 2008).

![Graphs showing Scale Factor (\( \lambda \)) and Shape Parameter (\( k \)) vs. Activity Level (mHem)]
Activity level and the analytic probability density function

\[ f(x | k, \lambda, \mu, \sigma, c) = (1 - c) \frac{k}{\lambda} \left( \frac{x}{\lambda} \right)^{k-1} e^{-\left( \frac{x}{\lambda} \right)^k} + \frac{c}{x\sigma \sqrt{2\pi}} e^{-\frac{(\ln x - \mu)^2}{2\sigma^2}} \]

- The Weibull’s parameters are not correlated with activity level (Hagenaar et al. 2003, 2008).
- The Log-Normal parameters correlate strongly with activity level

\[ \rho = 0.91, P > 0.99 \]
\[ \rho = 0.88, P > 0.99 \]
\[ \rho = 0.65, P = 0.99 \]
Activity level and the analytic probability density function

\[
f(x \mid k, \lambda, \mu, \sigma, c) = (1 - c) \frac{k}{\lambda} \left(\frac{x}{\lambda}\right)^{k-1} e^{-\left(\frac{x}{\lambda}\right)^{k}} + \frac{c}{x\sigma\sqrt{2\pi}} e^{-\frac{(\ln x - \mu)^2}{2\sigma^2}}
\]

- The Weibull’s parameters are not correlated activity level (Hagenaar et al 2003, 2008).
- The Log-Normal parameters correlate strongly with activity level.
- Cycle-dependent variations of the size-flux distribution are dictated exclusively by the large BMRs!
IMPLICATIONS

When I submitted my abstract I thought I understood what these results meant.
This is not the case anymore
In our first paper we speculated that the structures associated with each component were associated with different forms of dynamo action.
• Sunspot groups and pores are only observed in active latitudes (spatial dependence on cycle evolution).

• Why are the statistical properties of pores fixed?
• Sunspot groups and pores are only observed in active latitudes (spatial dependence on cycle evolution).

• Why are the statistical properties of pores fixed?
  • Self-similarity driven by convection?
  • Separate originating mechanisms?
Concluding Remarks

• The solar size-flux distribution is a composite of Weibull and log-normal distributions. A very simple modulation of its parameters captures cycle dependence.

• Only the parameters that characterize the log-normal distribution change with activity level.

• Our results seem robust and significant, but I have little understanding of the underlying mechanisms.

• Analysis of magnetic data, and of MHD simulations will be critical for furthering our understanding.